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A NEW POTENTIAL OF MILKY WAY GIVEN ANALYTICALLY

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Abstract. A mass distribution model for the Milky Way is presented. The potential of this galaxy is contributed by four subsystems: bulge, innermost dark matter, disc and outer dark matter. For each of them the potential is given analytically by using elementary functions. The values of the model parameters are specified by fitting an assumed rotation curve. The model is foreseen to be used in the determination of galactocentric orbits.

1. INTRODUCTION

The Galaxy structure is difficult to discern since we are observing from within. Traditionally, it has three components bulge, probably with bar, disc and dark halo (Dehnen & Binney, 1998). Each component contributes to gravitational field. A new idea was introduced by Iocco et al. (2015) where they showed that current observational data strongly disfavour baryons as the sole contribution to the Galactic mass budget, even inside the solar circle. In this paper we will show new potential of Milky Way consisting of four components, where innermost dark matter is the newest addition, together with new potentials for some components.

2. COMPONENTS OF THE MILAKY WAY

In order to construct a gravitational potential of the Milky Way analyti-cally, we restrict ourselves to axisymmetric models of components of the Milky Way and steady state. Also we use only elementary functions for describing the potential. As for the number of components, we use four in this study. Three main components being bulge, disc and dark corona, as in many previous studies, and the fourth one innermost dark matter. We use completely new formulas for disc and dark corona, that were recently published (Ninković, 2015; Ninković, 2017).

2.1. Innermost dark matter and bulge

For innermost dark matter (IDM) we use potentials of some simple systems, nevertheless when combined with other components of the Milky way, we get good results on final rotation curve, especially since it is known that many previous models when compared to measurements from the inner few kpc of Galaxy do a poor job of reproducing them. The potential of IDM inside limiting radius r_{IDM} is modelled as homogenous sphere with constant density ρ and mass $M_{IDM} = \frac{4}{3}\pi\rho r_{IDM}^3$, so we have that innermost dark matter contribution to circular speed (in km^2s^{-2}) is:

$$u_{cIDM} = \sqrt{\frac{GM_{IDM}}{r_{IDM}^3}} R, R\, r_{IDM}, \tag{1}$$

where *G* is the universal gravitation constant and R is variable, distance to the axis of symmetry (*z*). Outside the homogenous sphere we use Keplerian or point mass potential:

$$u_{cIDM} = \sqrt{\frac{GM}{R}}, R > r_{IDM}. \tag{2}$$

In case of a spherically symmetric subsystem, such as bulge (B), the generalised isochrones potential formula is often used that was proposed in 1973 (Kuzmin & Veltmann, 1973):

$$\Pi_B = \frac{GM_B}{r_a + \sqrt{r_b^2 + r^2}}. \tag{3}$$

Here Π is potential, M_B is total mass of bulge, *r* is the distance to the Galactic centre and r_a and r_b are constants with dimension of length. Finally, we have bulge contribution to circular speed:

$$u_{cB} = \sqrt{\frac{GM_B R^2}{\sqrt{r_b^2 + R^2}\left(r_a + \sqrt{r_b^2 + R^2}\right)^2}}. \tag{4}$$

2.2. Disc

The potential of the flattened stellar subsystem, especially disc (D) is often represented by Miyamoto-Nagai (1975) formula:

$$\Pi = \frac{GM_D}{R_{MN}},$$

(5)

$$R_{MN} = \sqrt{R^2 + \left(a + \sqrt{z^2 + b^2}\right)^2}.$$

Here potential is a function of two arguments R and z, while a and b are constants. In recent study done by Ninković (2015) a new potential formula applicable to flattened system is presented. It is a modification of the Miyamoto-Nagai potential and can easily be applied to exponential discs. It consists of a new term R_N, which is also function of the same variables, added in denominator of eq. (5).

$$\Pi_D = \frac{GM_D}{R_{MN} - R_N}.$$

(6)

This new term introduce new constants, disc scale length R_d and dimensionless c_1, c_2 and γ_1, γ_2.

$$R_N = \frac{1}{2} R_d \left[\left(1 + \frac{R^2}{c_1^2}\right)^{\gamma_1} + \left(1 + \frac{z^2}{c_2^2}\right)^{\gamma_2}\right].$$

(7)

For purpose of creating new four-component potential of Milky Way we adopted this formula for disc. Then disc contribution to circular speed is:

$$u_{cD} = \sqrt{\frac{GM_D R}{\left[\sqrt{R^2 + (a+b)^2} - \frac{1}{2} R_d \left(1 + \left(1 + \frac{R^2}{R_d^2}\right)^{\gamma_1}\right)\right]^2}}.$$

(8)

$$\cdot \sqrt{\frac{r}{\sqrt{R^2 + (a+b)^2}} - \gamma_1 \frac{R}{R_d}\left(1 + \frac{R^2}{R_d^2}\right)^{\gamma_1 - 1}}.$$

2.3. Dark corona

For density of dark corona (DC) for this model, we use formula (5) from paper (Ninković, 2017) which was the first time such formula was published, and its reproduced here:

$$\rho_{DC} = \rho_0 \left(\frac{1}{1 + \xi^k} - \frac{1}{1 + \xi_l^k}\right).$$

(9)

Case $k = 3$ is analysed in details in original paper and we chose this as best option for our model. From (9) it follows that cumulative mass for DC is:

$$M_{DC} = 4\pi\rho_0 r_c^3 \left[\frac{ln(1+\xi^3)}{3} - \frac{\xi^3}{3(1+\xi_l^3)}\right]. \quad (10)$$

In eq. (9) k is supposed to be a natural number and for practical reasons $2 \leq k \leq 4$. In eq. (9) and (10) r_c is corona scale length, r_l is corona limiting radius, $\xi = \frac{r}{r_c}$, $\xi_l = \frac{r_l}{r_c}$. As seen from eq.(9), at $\xi = \xi_l$ we get $\rho = 0$, and if $\xi > \xi_l$ then $\rho = 0$. Thus M_{DC} is equal to total mass for $\xi = \xi_l$. Now we can obtain the potential for DC:

$$\Pi = \frac{GM_{DC}}{R} + 4\pi G\rho_0 r_c^2 \left\{\frac{1}{6} ln \frac{\frac{4}{3}\left(\xi_l - \frac{1}{2}\right)^2 + 1}{\frac{4}{3}\left(\xi - \frac{1}{2}\right)^2 + 1} + \frac{\sqrt{3}}{3} arctg\left[\frac{2\sqrt{3}}{3}\left(\xi_l - \frac{1}{2}\right)\right] - \frac{\sqrt{3}}{3} arctg\left[\frac{2\sqrt{3}}{3}\left(\xi - \frac{1}{2}\right)\right] - \frac{1}{3} ln \frac{\xi_l + 1}{\xi + 1} - \frac{\xi_l^2 - \xi^2}{2(1+\xi_l^3)}\right\}. \quad (11)$$

In view of eq. (10) the circular speed of the dark corona is:

$$u_{cDC} = \sqrt{\frac{GM_{DC}}{R}}. \quad (12)$$

3. RESULTS

In order to obtain values for all free parameters in our models of components of Milky Way we compare our results with up-to-date compilation of Milky Way rotation curve measurements presented in the paper (Iocco et al, 2015). The data points on the Figure 1. are taken from mentioned paper, while three lines overplotted on the graph are our best fit models of the rotation curve measurements. Each model, with all four components shown, is also presented separately on Figure 2. These models are obtained using four-component analytical model discussed in previous section. This is first time such model is used and we show that fourth component improves our fit especially in inner part of the Milky Way where all previous models fail to successfully fit observational data. All parameters for these three models are presented in Table 1.

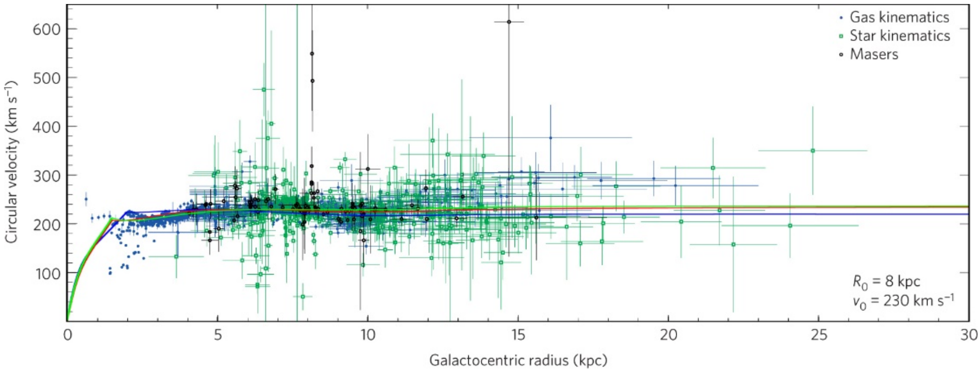

Figure 1: Data points are taken from (Iocco et al, 2015) while three overplotted lines are our best fit models of Milky Way rotation curve. Parameters for these models are given in Table 1.

We have discussed two variants concerning the values for $R_{\odot}$ and $u_c(R_{\odot})$, mentioned very often in the literature: $R_{\odot} = 8.5$ kpc, $u_c(R_{\odot}) = 220$ km/s and $R_{\odot} = 8.0$ kpc, $u_c(R_{\odot}) = 230$ km/s where the latter one corresponds to the values given by Iocco et al. (2015) for comparison.

In the case of innermost dark matter region, we found that the best fit for inner part of Milky Way rotation curve is produces for $7 < M_{IDM} < 10$ billion solar masses and limiting radius for homogenous sphere $1.5 \leq r_{IDM} \leq 2$ kpc.

Table 1: Parameters that were used in all models. Units: Total mass of each component $M \times 10^9 M_{\odot}$; ρ_0 in $M_{\odot} pc^{-3}$; $r_{IDM}, r_a, R_d, R_{\odot}$ in kpc; $u_c(R_{\odot})$ in km/s.

		Model 1 Blue line	Model 2 Green line	Model 3 Red line
$R_{\odot}$		8.5	8.0	8.0
$u_c(R_{\odot})$		220	230	227
Innermost dark matter (IDM)	M_{IDM}	9.5	8	7.0
	r_{IDM}	2	1.5	1.5
Bulge (B)	M_B	10	8.5	7.5
	r_a	0.5	0.5	0.5
Disc (D)	M_D	58	69	66
	R_d	2.85	3.2	2.9
	γ_1	-0.1	-0.05	-0.1
Dark corona (DC)	ρ_0	0.006	0.007	0.007

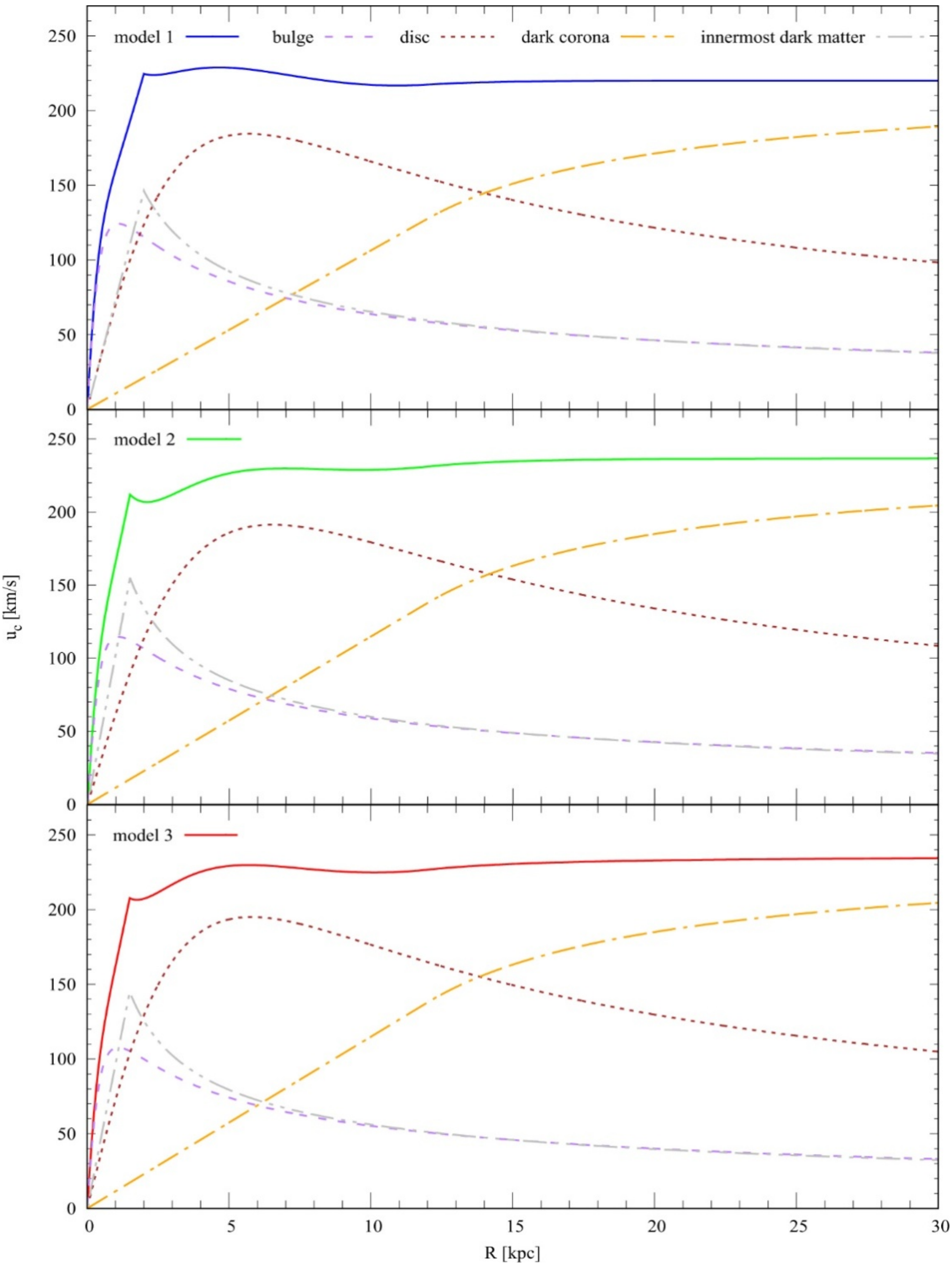

Figure 2: Rotation curve for models presented with contribution from each of four components separately.

For bulge total mass we used a few different values starting from 7.5 up to 10 billion solar masses. In this case we used that $r_a = r_b$ (eq. (3)) and using accordingly formulae for total mass and density, it is easily obtained that within r_a, about 12% of the total mass is contained and that the ratio of the density at the centre to that at r_a is about 3.23.

When it comes to Galactic disc, starting value is surface density $\sigma(R_\odot)$ where $R_\odot$ is galactocentric distance of the Sun. For surface density, we take a value well known in literature $\sigma(R_\odot) = 50M_\odot$ pc^{-2}. In eq. (7) it is also acceptable to substitute c_1 with R_d. In fitting the rotation curve we also determined that best value for ratio $\frac{a+b}{R_d}$ is 2.1. Since this component is flattened, it must satisfy $a \gg b$. For disc scale length we tested interval $2.5 < R_d \leq 3.5$ Exponent γ_1 cannot exceed 0.5 and should be small negative number. Total mass of disc M_D is varied between 50 and 70 billion solar masses.

For dark corona, one should first decide on parameter k, since it should be in interval $[2,4]$. If $k = 3$ is chosen then total mass will depend only on three parameters: ρ_0, r_c and r_l. We find this to be best option when dark corona is modelled. Formula (11) is applied to the circle outside $R = R_\odot$ and we assumed fixed values for $r_c = 2R_\odot$ kpc ($\xi = 0.5$) and $\xi_l = 5.665$.

Limiting case for total mass of Milky Way galaxy was constrained by recent finds of Patel et al. (2018). They showed, by using the Bayesian framework to include all Milky Way satellites with measured 6D phase space information and applying it with the Illustris-Dark simulation, that the Galaxy's total mass should be constrained to $0.85^{+0.23}_{-0.26} \times 10^{12} M_\odot$.

4. CONCLUSION

When Milky Way mass models are discussed most often there are three main contributors to the potential: the bulge, the disc and dark corona. Here we presented a new model with forth contributor innermost dark matter. We show that this could be a missing part in order to better fit rotation curve of Milky Way to most recent observational data, especially in the inner part of Milky Way. For all of them steady state has been assumed and potentials are all given analytically by using only elementary functions. This gives big advantage in making numerical integrator for calculating galactocentric orbits of stars. Such integrators work fast and can handle huge databases which is important in light of new results given by GAIA mission.

Innermost dark matter region is a new component that we introduced here so we used simplest model. The potential of IDM is represented by homogenous sphere. Bulge and disc are observable components of Milky Way and as such these components are best analysed and potential of these components are most complicated. Bulge is represented by generalised isochrones potential formula which is often used, while for the disc we used completely new exponential formula given by Ninković. It is modification of well-known Miyamoto-Nagai formula. Modelling of dark corona is harder since only constraints due to seen matter can be used. Here we use just one case of more complicated new formula given by Ninković recently. This formula should satisfy requirement that there is a prominent maximum near the centre due to the bulge model, but also provides high values for the circular speed in outer parts due to the dark matter model being accepted here.

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